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Solution Manual for

Robotics
Modelling, Planning and Control

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Solutions to analytical problems are developed by emphasizing the crucial steps towards the solution. Some problems may be solved in different ways; the solution reported in the manual is believed to be the most straightforward. The solutions to several problems contain useful analytical developments which are complementary to the theoretical derivation in the textbook.

Solutions to programming problems are accompanied by results of computer implementations in MATLAB® (version 7.4) with SIMULINK®. The code (downloadable from www.springer.com/978-1-84628-641-4) is available free of charge to those adopting this volume as a text for courses.

The software is not aimed at providing a complete toolbox, but only at solving the end-of-chapter problems. Nonetheless, the code has been developed in a modular fashion which should allow direct expansion to more complex problems as well as ease of changing the problems data.

For the problems solved in MATLAB, the solution is contained in a file with .m extension, where the first letter is an s, followed by the problem number, e.g., s4_1.m is the file to execute for solving Problem 4.1.

The problems requiring simulation of a dynamic system have been solved in SIMULINK and the solution is contained in a file with .mdl extension, e.g., s3_21.mdl is the file to execute for solving Problem 3.21. Each problem of this kind requires the initialization of certain variables before starting the simulation. This is performed in a file where the first letter is an i, followed by the problem number, e.g., i3_21.m is the initialization file for Problem 3.21.

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1 MATLAB and SIMULINK are registered trademarks of The MathWorks, Inc.
For both MATLAB- and SIMULINK-based problems, the output plots of relevant variables are obtained by executing a file where the first letter is a p, followed by the problem number, e.g., p3_21.m is the file for plotting the output variables of Problem 3.21.

Variable initialization and plot can be activated by double clicking respectively on the upper-left block and the lower-right block in the SIMULINK block diagrams.

For problems requiring the simulation of two different systems, two files have been created where letters a and b have been used to distinguish them, e.g., s3_22a.mdl and s3_22b.mdl are the files for solving Problem 3.22 with two different algorithms; accordingly, the files for plotting output variables have been named p3_22a.m and p3_22b.m.

The above files are supplemented by other function and script files which are needed to solve the programming problems.

All the files used to solve a given problem are collected into a folder with the same label of the problem, e.g. Folder 3_21 contains the files of Problem 3.21.

Helpful comments are added to each file to describe its contents and functions. A readme.txt file is also provided.

Finally, the authors wish to thank Luigi Freda for his contributions to the software developed for the solution of some problems of Chapter 12.

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Solution to Problem 2.1

Composition of rotation matrices with respect to the current frame gives

\[ R(\phi) = R_z(\varphi)R_{x'}(\vartheta)R_{z''}(\psi). \]

Using the expressions of elementary rotation matrices in (2.6) and (2.8):

\[
R_z(\varphi) = \begin{bmatrix}
c_\varphi & -s_\varphi & 0 \\
s_\varphi & c_\varphi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
R_{x'}(\vartheta) = \begin{bmatrix}
1 & 0 & 0 \\
0 & c_\vartheta & -s_\vartheta \\
0 & s_\vartheta & c_\vartheta
\end{bmatrix}
\]

\[
R_{z''}(\psi) = \begin{bmatrix}
c_\psi & -s_\psi & 0 \\
s_\psi & c_\psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

and taking the products gives

\[
R(\phi) = \begin{bmatrix}
c_\varphi c_\psi - s_\varphi s_\psi c_\vartheta & -c_\varphi s_\psi - s_\varphi c_\vartheta c_\psi & s_\varphi s_\vartheta \\
-s_\varphi c_\psi + c_\varphi s_\psi c_\vartheta & -s_\varphi s_\psi + s_\varphi c_\psi c_\vartheta & -c_\varphi c_\vartheta \\
-s_\vartheta s_\psi & s_\varphi c_\psi & c_\vartheta
\end{bmatrix}.
\]

As for the inverse problem, given a rotation matrix

\[
R = \begin{bmatrix}
r_{11} & r_{12} & r_{13} \\
r_{21} & r_{22} & r_{23} \\
r_{31} & r_{32} & r_{33}
\end{bmatrix},
\]
2 Kinematics

the set of Euler angles ZXZ is given by

\[
\begin{align*}
\varphi &= \text{Atan2}(r_{13}, -r_{23}) \\
\theta &= \text{Atan2}\left(\sqrt{r_{31}^2 + r_{32}^2}, r_{33}\right) \\
\psi &= \text{Atan2}(r_{31}, r_{32})
\end{align*}
\]

when \( \theta \in (0, \pi) \). Otherwise, if \( \theta \in (-\pi, 0) \) then the solution is

\[
\begin{align*}
\varphi &= \text{Atan2}(-r_{13}, r_{23}) \\
\theta &= \text{Atan2}\left(-\sqrt{r_{31}^2 + r_{32}^2}, r_{33}\right) \\
\psi &= \text{Atan2}(-r_{31}, -r_{32}).
\end{align*}
\]

Solution to Problem 2.2

In the case \( s_\psi = 0 \), the rotation matrix in (2.18) becomes

\[
R(\phi) = \begin{bmatrix}
c_{\phi+\psi} & -s_{\phi+\psi} & 0 \\
s_{\phi+\psi} & c_{\phi+\psi} & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

when \( \theta = 0 \). Otherwise, if \( \theta = \pi \), then the matrix is

\[
R(\phi) = \begin{bmatrix}
-c_{\phi-\psi} & -s_{\phi-\psi} & 0 \\
s_{\phi-\psi} & c_{\phi-\psi} & 0 \\
0 & 0 & 1
\end{bmatrix}.
\]

From the elements [1, 2] and [2, 2] it is possible to compute only the sum or difference of angles \( \phi \) and \( \psi \), i.e.,

\[
\varphi \pm \psi = \text{Atan2}(-r_{12}, r_{22})
\]

where the positive sign holds for \( \theta = 0 \) and the negative sign holds for \( \theta = \pi \).

Solution to Problem 2.3

In the case \( c_\psi = 0 \), the rotation matrix in (2.21) becomes

\[
R(\phi) = \begin{bmatrix}
0 & s_{\psi-\phi} & c_{\psi-\phi} \\
0 & c_{\psi-\phi} & -s_{\psi-\phi} \\
-1 & 0 & 0
\end{bmatrix}
\]

when \( \theta = \pi/2 \). Otherwise, if \( \theta = -\pi/2 \), then the matrix is

\[
R(\phi) = \begin{bmatrix}
0 & -s_{\psi+\phi} & -c_{\psi+\phi} \\
0 & c_{\psi+\phi} & -s_{\psi+\phi} \\
1 & 0 & 0
\end{bmatrix}.
\]
From the elements $[2, 2]$ and $[2, 3]$ it is possible to compute only the sum or difference of angles $\psi$ and $\varphi$, i.e.,

$$\psi \pm \varphi = \text{Atan2}(-r_{23}, r_{22})$$

where the positive sign holds for $\vartheta = -\pi/2$ and the negative sign holds for $\vartheta = \pi/2$.

**Solution to Problem 2.4**

The rotation matrix can be obtained as in (2.24)

$$R(\vartheta, r) = R_z(\alpha)R_y(\beta)R_z(\vartheta)R_y(-\beta)R_z(-\alpha),$$

where the elementary rotation matrices are given as in (2.6) and (2.7):

$$R_z(\alpha) = \begin{bmatrix} c_\alpha & -s_\alpha & 0 \\ s_\alpha & c_\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$R_y(\beta) = \begin{bmatrix} c_\beta & 0 & s_\beta \\ 0 & 1 & 0 \\ -s_\beta & 0 & c_\beta \end{bmatrix},$$

$$R_z(\vartheta) = \begin{bmatrix} c_\vartheta & -s_\vartheta & 0 \\ s_\vartheta & c_\vartheta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$  

Taking the first product gives

$$R_z(\alpha)R_y(\beta) = \begin{bmatrix} c_\alpha c_\beta & -s_\alpha c_\beta & c_\alpha s_\beta \\ s_\alpha c_\beta & c_\alpha c_\beta & s_\alpha s_\beta \\ -s_\beta & 0 & c_\beta \end{bmatrix}.$$

The next product gives

$$R_z(\alpha)R_y(\beta)R_z(\vartheta) = \begin{bmatrix} c_\alpha c_\beta c_\vartheta - s_\alpha s_\vartheta c_\beta - s_\alpha c_\vartheta s_\beta + c_\alpha c_\beta s_\vartheta & c_\alpha c_\beta s_\vartheta + s_\alpha c_\beta c_\vartheta & -c_\alpha s_\beta \\ s_\alpha c_\beta c_\vartheta + c_\alpha s_\vartheta c_\beta - s_\alpha c_\vartheta s_\beta + c_\alpha c_\beta s_\vartheta & s_\alpha c_\beta s_\vartheta - c_\alpha c_\beta c_\vartheta & c_\alpha c_\beta s_\vartheta + s_\alpha c_\beta c_\vartheta \\ -s_\beta c_\vartheta & s_\beta s_\vartheta & c_\beta \end{bmatrix}.$$  

Then, by observing that

$$R_y(-\beta)R_z(-\alpha) = (R_z(\alpha)R_y(\beta))^T,$$

the overall rotation matrix is

$$R(\vartheta, r) = \begin{bmatrix} (s^2_\alpha + c^2_\alpha c^2_\beta) c_\vartheta + c^2_\alpha s^2_\beta(1 - c_\vartheta) - c_\beta s_\vartheta \\ s_\alpha c_\alpha s^2_\beta(1 - c_\vartheta) + c_\beta s_\vartheta & (s^2_\alpha c^2_\beta + c^2_\alpha c^2_\vartheta) c_\vartheta + s^2_\alpha s^2_\beta \\ c_\alpha s_\beta c_\vartheta(1 - c_\vartheta) - s_\alpha s_\beta s_\vartheta & s_\alpha s_\beta c_\vartheta(1 - c_\vartheta) + c_\alpha s_\beta s_\vartheta \\ s_\alpha s_\beta c_\vartheta(1 - c_\vartheta) - c_\alpha s_\beta s_\vartheta & s^2_\beta c_\vartheta + c^2_\beta \end{bmatrix}.$$
Recalling the relations
\[ s_\alpha = \frac{r_y}{\sqrt{r_x^2 + r_y^2}}, \quad c_\alpha = \frac{r_x}{\sqrt{r_x^2 + r_y^2}} \]
\[ s_\beta = \sqrt{r_x^2 + r_y^2}, \quad c_\beta = r_z \]
\[ r_x^2 + r_y^2 + r_z^2 = 1 \]
and using the following identities:
\[ s_\alpha^2 + c_\alpha^2 = 1 - r_x^2 \]
\[ s_\alpha^2 c_\beta^2 + c_\alpha^2 = 1 - r_y^2 \]
leads to expression (2.25).

**Solution to Problem 2.5**

From the expression of \( R(\vartheta, r) \) in (2.25), summing the elements on the diagonal gives
\[ r_{11} + r_{22} + r_{33} = 1 + 2 \cos \vartheta \]
from which the angle is
\[ \vartheta = \cos^{-1} \left( \frac{r_{11} + r_{22} + r_{33} - 1}{2} \right). \]

Then, taking suitable differences between the off-diagonal elements gives
\[ r_{32} - r_{23} = 2r_z \sin \vartheta \]
\[ r_{13} - r_{31} = 2r_y \sin \vartheta \]
\[ r_{21} - r_{12} = 2r_x \sin \vartheta \]
and thus
\[ r = \frac{1}{2 \sin \vartheta} \begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}. \]

In the case \( \sin \vartheta = 0 \), if \( r_{11} + r_{22} + r_{33} = 3 \), then \( \vartheta = 0 \); this means that no rotation has occurred and \( r \) is arbitrary. Instead, if \( r_{11} + r_{22} + r_{33} = -1 \), then \( \vartheta = \pi \) and the expression of the rotation matrix becomes
\[ R(\pi, r) = \begin{bmatrix} 2r_x^2 - 1 & 2r_x r_y & 2r_x r_z \\ 2r_x r_y & 2r_y^2 - 1 & 2r_y r_z \\ 2r_x r_z & 2r_y r_z & 2r_z^2 - 1 \end{bmatrix}. \]